Algorithms and Heuristics for Deployment of Sensors (“Guards”) for Optimal Coverage

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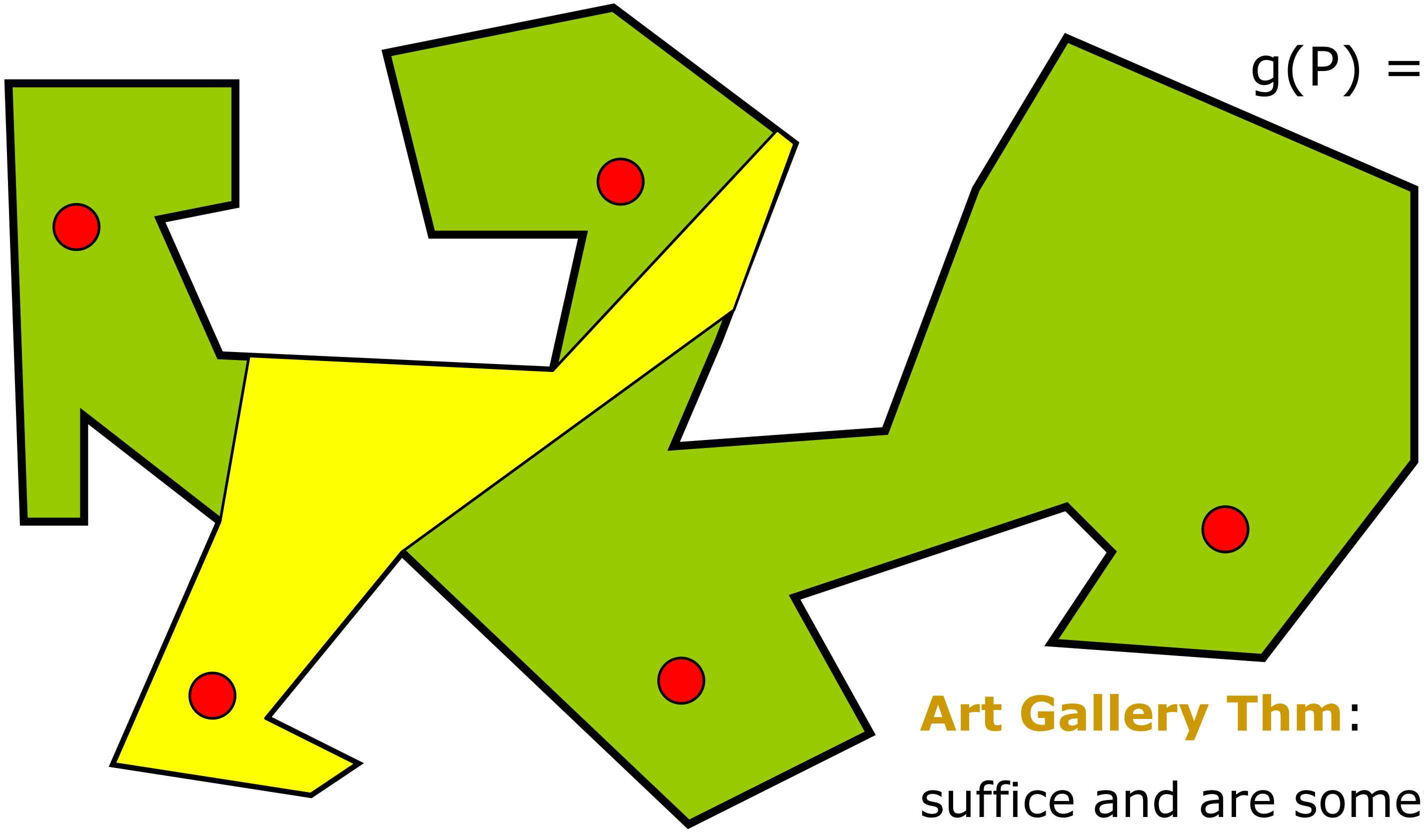
Stationary Guards

The Art Gallery Problem

Determine a small set of "guards" to see all of a given n-vertex polygon P

NP-hard, even in simple polygon

g(P) = min # guards



Art Gallery Thm: $\lfloor \frac{n}{3} \rfloor$ guards suffice and are sometimes necessary

Motivation: Sensor coverage, security

Experimental Investigation [Amit, M, Packer]

- Propose several heuristics for computing guards

- Experimental analysis and comparison

- Compute both upper bounds and lower bounds on OPT, so we can bound how close to OPT we get

- Conclude: heuristics work well in practice:
  - Either find OPT solution or close to optimal
  - Almost always 2-approx
    (always for “random” polygons)

Related Work

- Combinatorics: Lots!

Art Gallery Thm: $\lfloor \frac{n}{3} \rfloor$ guards suffice and are sometimes necessary

- Approximation algorithms for discrete candidate sets (vertex guards, grid-point guards, etc):
  - O(log n)-approx: set cover (greedy) [G87]
  - O(log k)-approx: reweighting ([Cl,BG]) [EH03,GL01]
  - O(1)-approx in special cases:
    - 1.5D terrains (best: 4-approx) [BKM05,K06,EKMMS08]
    - Monotone polygons [Ni05]
- Pseudo-poly O(log k)-approx (poly in spread, n) [DKDS07]
- Exact poly-time solutions:
    - Rectangle visibility in rectilinear polygons [WK06]
    - *Partitioning* P into min # star-shaped pieces [Ke85]
    - Min-length watchman tour (mobile guard) [CN86]
- Other recent experiments
  - Experiments with (exp-time) combinatorial algorithm for guarding the boundary of P [BL06]

Greedy Heuristics

- Two phases:
  - Generate a set of good *candidate* guard positions
  - Greedily select a subset of candidates that fully cover *P*

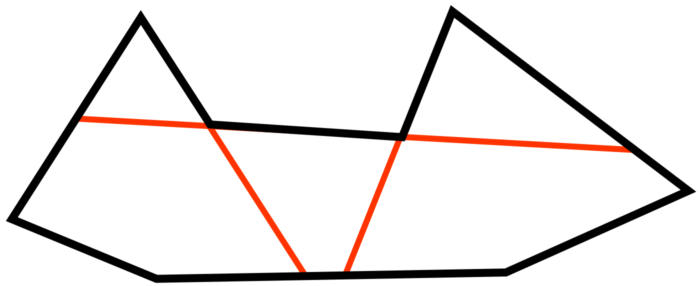
- Algorithm design choices:
  - How to specify the set of candidates?
  - How to score candidates for greedy selection?

Phase 1: Generating Candidates

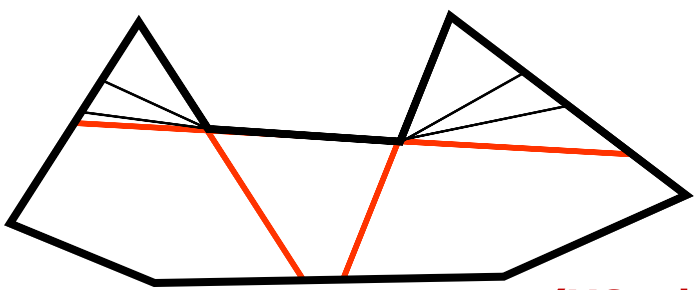
1. Use set V(P) = vertices of polygon P

(actually used points perturbed interior to P)

2. Centers C(P) of convex cells in an arrangement:
   - Edge extensions [size $O(n^2)$]



   - Visibility extensions [size $O(n^4)$]

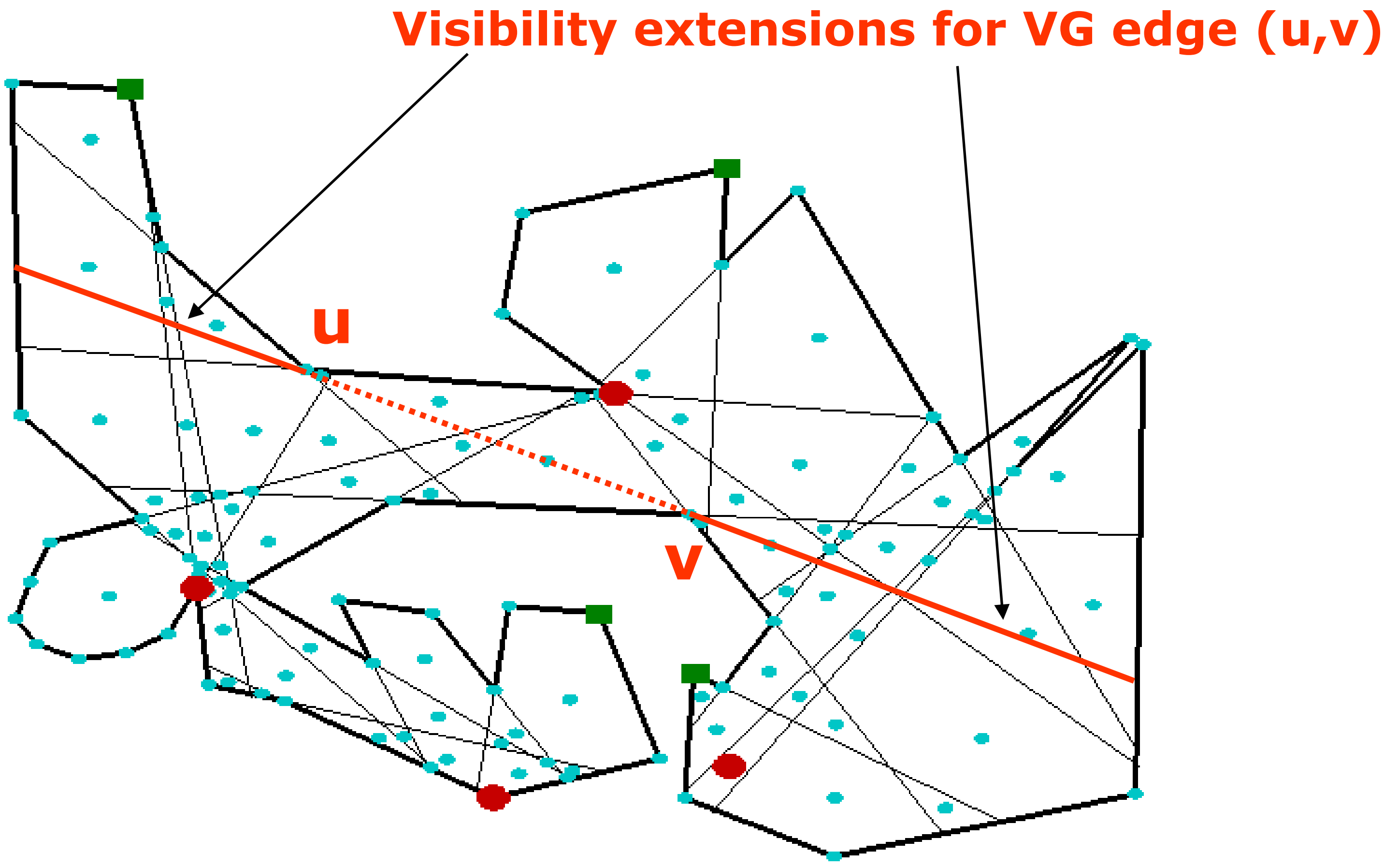


(VG edges incident on at least 1 reflex vertex)

3. $V(P) \cup C(P)$

Example

Centers of cells in arrangement of edge extensions



Phase 2: Greedily Selecting Candidates

- Set of candidates: $W(P)$

- Greedily add "good" candidates $g \in W(P)$ until P is covered: Max $\mu(g)$ $g \in W(P)$
- At end, iteratively remove redundant guards until set is *minimal*

Heuristics Used in Experimentation

- A_1 :

  Candidates W(P) = V(P) ∪ C(P)

  Vertices and center points in arr

  Score $\mu(g)$ = # unseen candidates

  Arrangement: Edge extensions

- A_2 :

  Variant: With each guard g chosen, add to arrangement the visibility edges V(g) induced by g

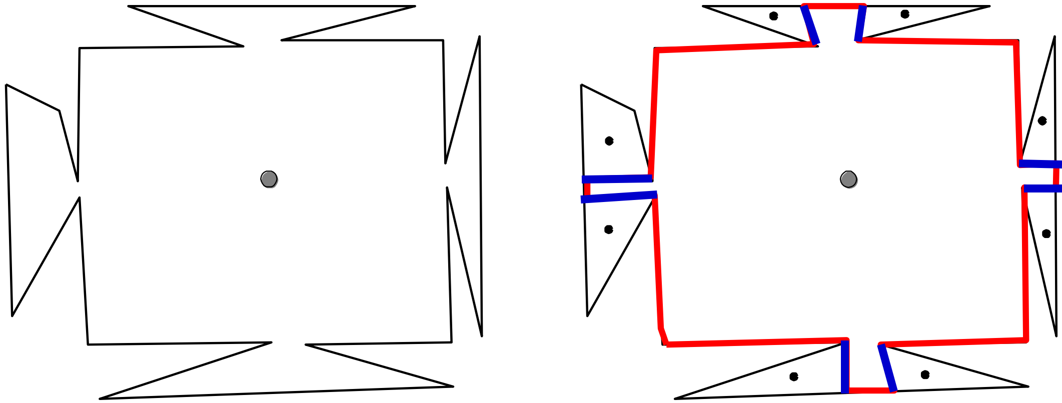


Figure 2: Using algorithm A_2: (a). The polygon and the first guard to be selected (shaded). (b). The visibility polygon of the guard (highlighted, in red) caused the addition of 8 new candidates (small black disks).

Blue: added edges

Heuristics Used in Experimentation

A_3 : Like A_1 but: Score $\mu(g)$ = *area* newly seen

A_4 : Like A_1 but: $\mu(g)$ weighted by *cell area*

A_5 : Like A_4 but: $\mu(g)$ weighted by shared bd(P)

A_6 : Like A_4 but: $\mu(g)$ weighted by % of shared bd(P)

A_7 : Like A_1 but: Candidates $W(P) = V(P)$

A_8 : Like A_1 but: Candidates $W(P) = C(P)$

A_9 : Like A_1 but: $\mu(g)$ = # newly seen vertices

A_{10} : Like A_1 but: $\mu(g)$ = # newly seen cell centers

A_{11} : Like A_1 but: Arrangement of *visibility extensions*

A_{12} : Combination of A_2 and A_{11}

(dynamically added edges, arr of visibility extensions)

Method: A_{13} : Probabilistic Reweighting

We also implemented an algorithm based on the Clarkson/Bronnimann-Goodrich framework: [EH03,GL01]

Each candidate is assigned a weight : probability it is selected

Initially: All weights = 1

Iteration: A candidate is selected at random

If there is an unguarded point, q, then the weights of candidates that see q are *doubled* **(improve chances q is guarded on future iterations)**

Continue until all points of P are guarded

Method: A_{14} : Polygon Partition

We also implemented an algorithm based on partitioning P into star-shaped pieces

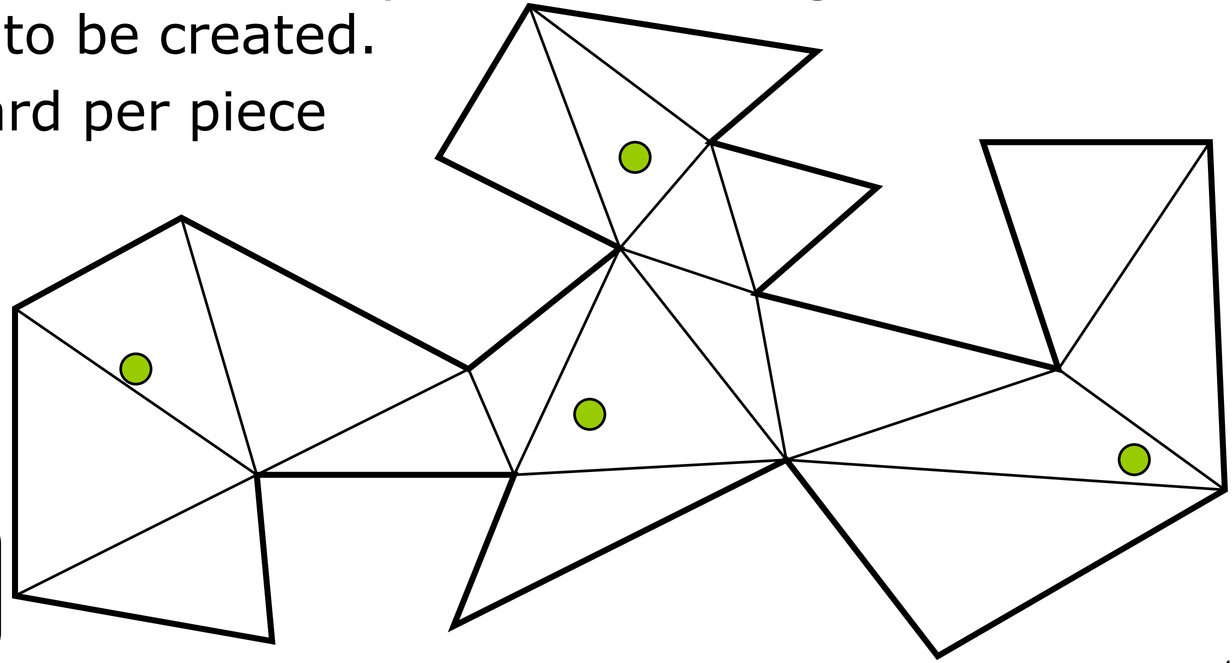
(Note: min-size partition into star-shaped polygons is poly-time, using DP)

We use a simple heuristic similar to Hertel-Mehlhorn 4-approx for min-cardinality convex partition:

- Triangulate P
- Remove diagonals iteratively, never allowing a non-star-shaped piece to be created.
- Place one guard per piece

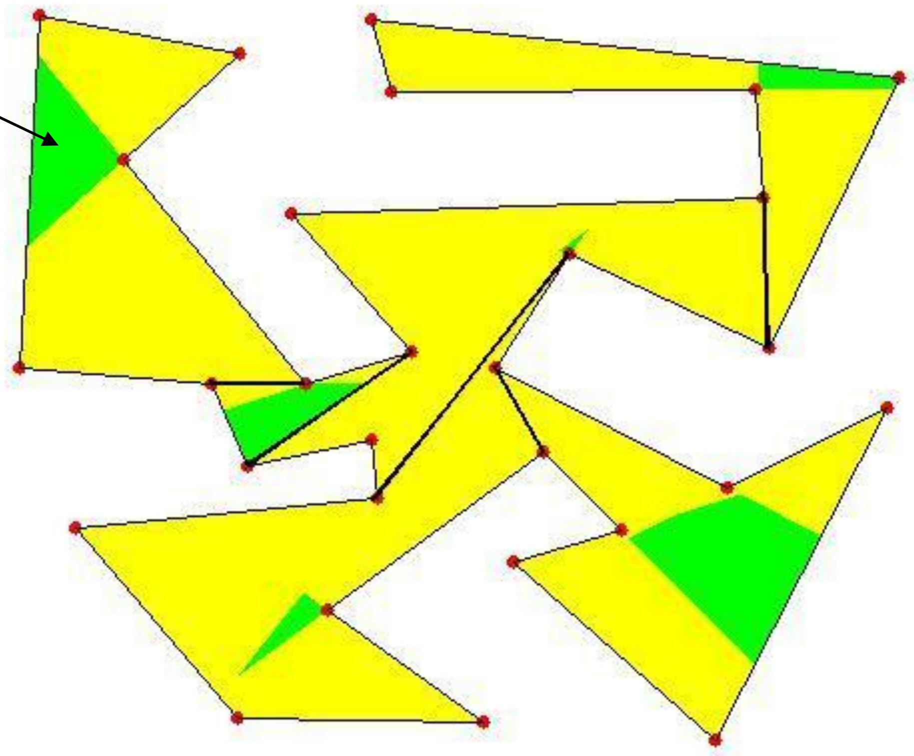
Not competitive with other methods (most cases)

Particularly poor on "spike box" examples



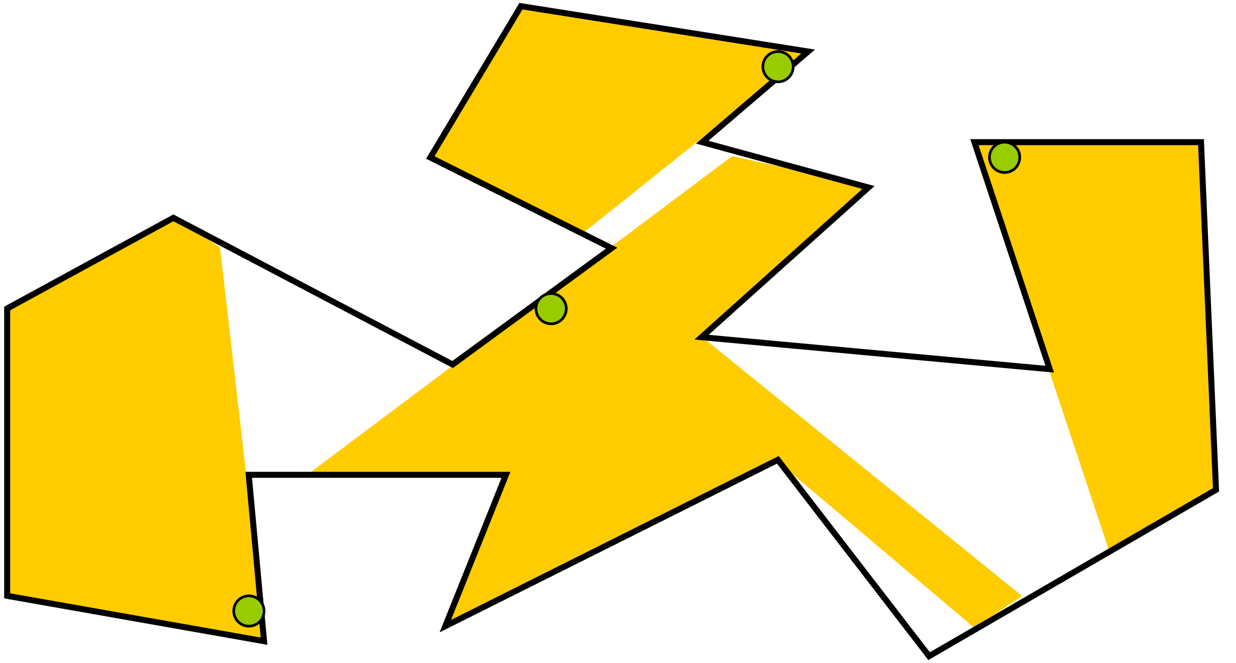
Example: A_{14} : Polygon Partition

kernels in green



Lower Bounds on OPT

Lemma: $g(P) \geq |I|$, for any visibility-independent set I of points in P



$g(P) \geq 4$

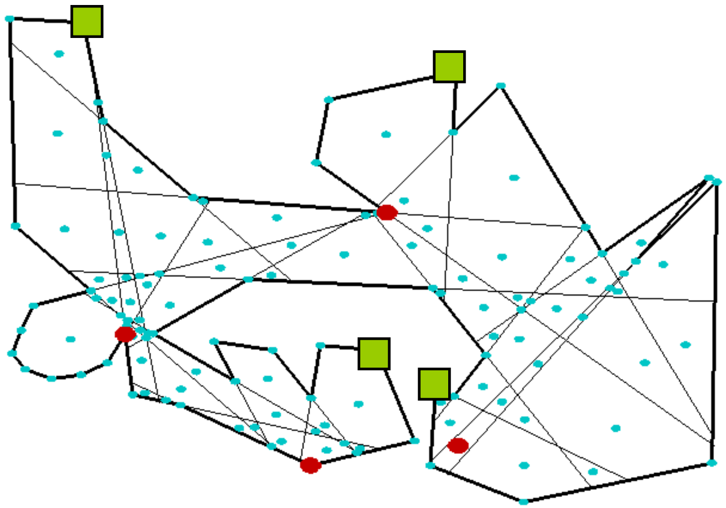
Lower Bounds on OPT

We greedily compute a visibility-independent set I:

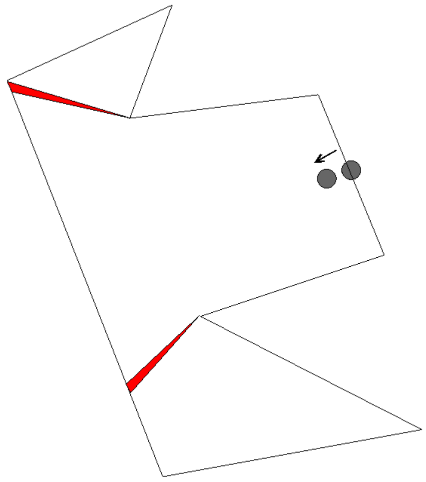
- Generate candidate set S   (not vis-indep)
- Add points $p \in S$ iteratively to I, minimizing # points of S seen by p, making sure that VP(p) is disjoint from VP(q), for $q \in I$

(We use CGAL arrangements to maintain VP's and test vis-independence)

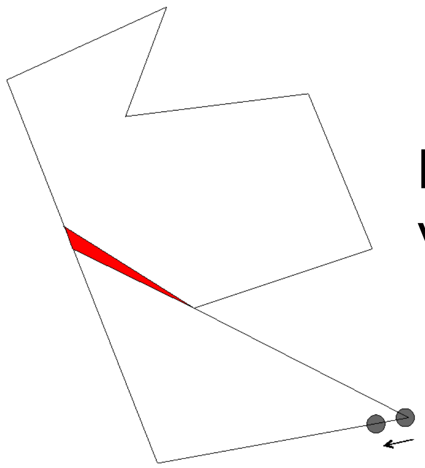
- Remove from S points seen by p
- Stop when S is empty



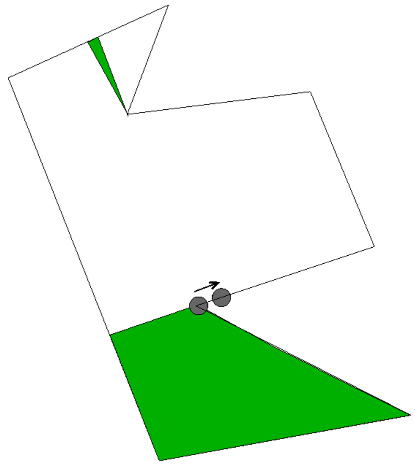
Lower Bounds on OPT



Most cases: $p \in bd(P)$ sees less



Moving away from a convex vertex tends to see more



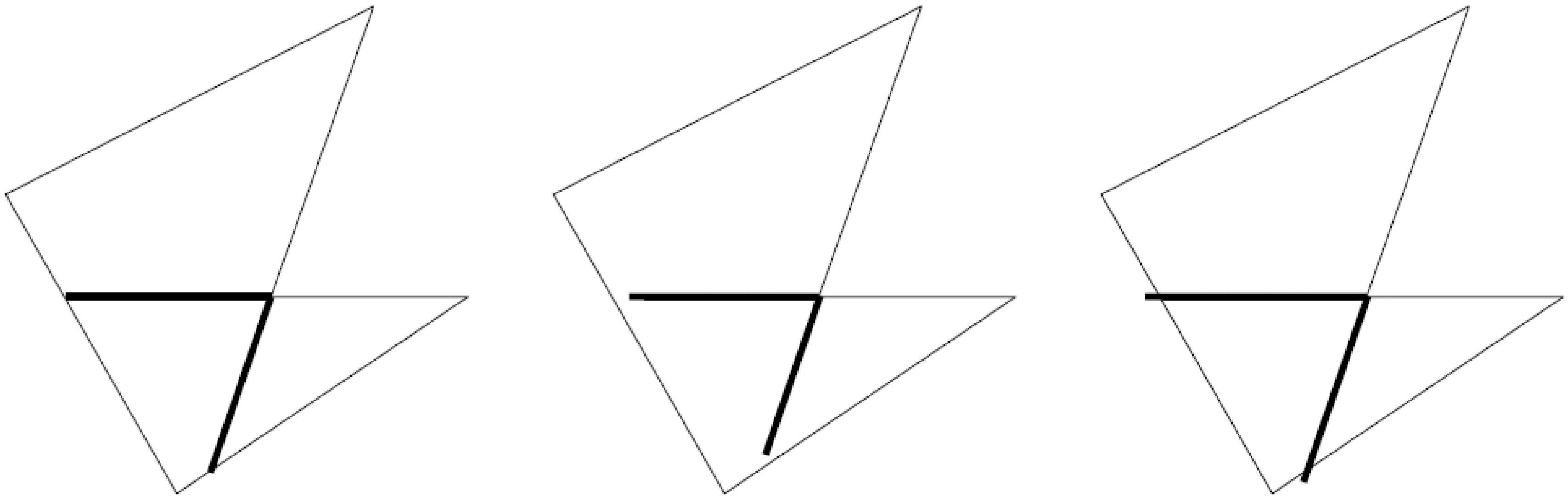
Moving away from a reflex vertex tends to see less

Heuristic: Candidates S are convex vertices and midpoints of edges of P joining two reflex vertices

Experimental Setup

- Windows XP, Pentium 4 (3.2GHz, 2.0GB)
- Visual .Net compiler; openGL; CGAL
- Randomly generated polygons:
  - RPG of Auer and Held, 50-200 vertices
- Manually generated special polygons

Robust computation of cells

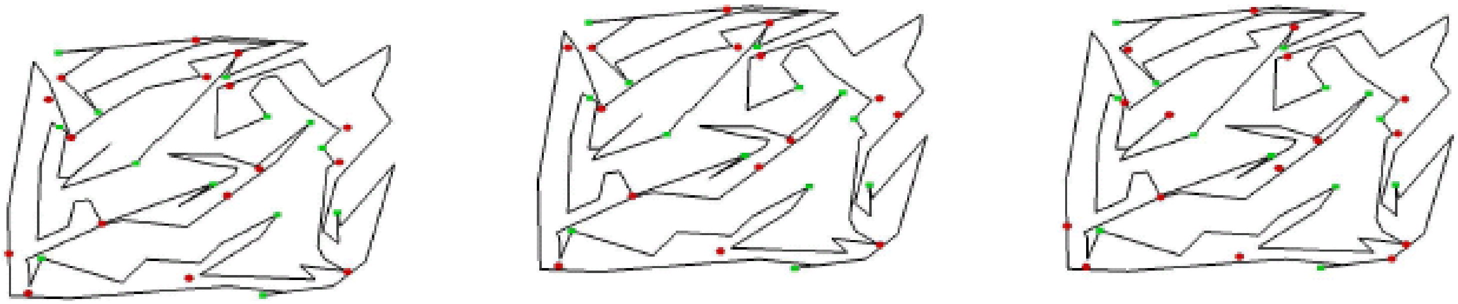


With exact arithmetic

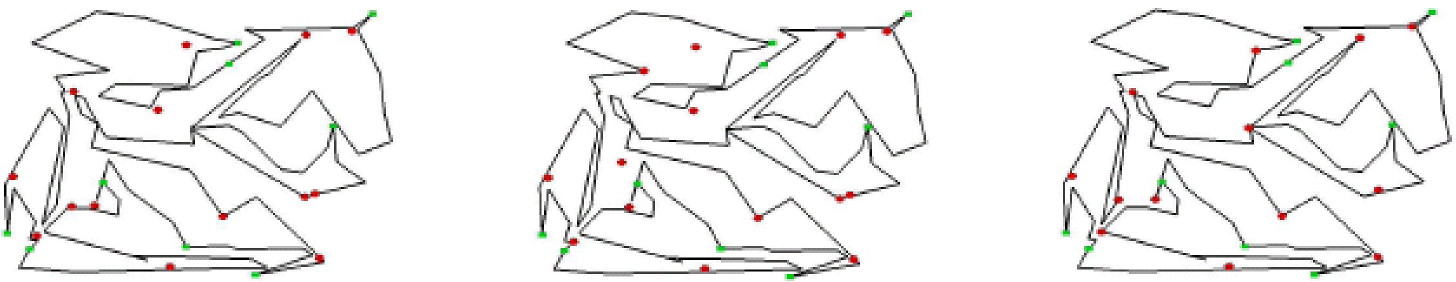
Possible error with floating-point

Solution: push extensions

Examples: n=100



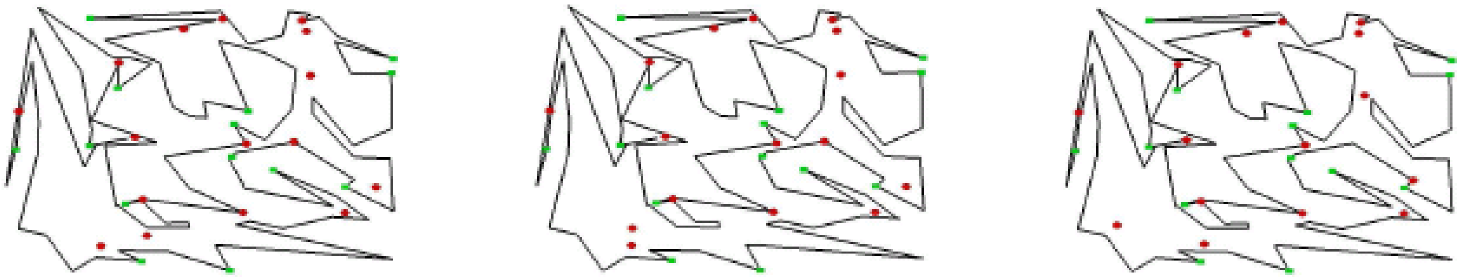
(a) 16 guards (b) 15 guards (c) 16 guards



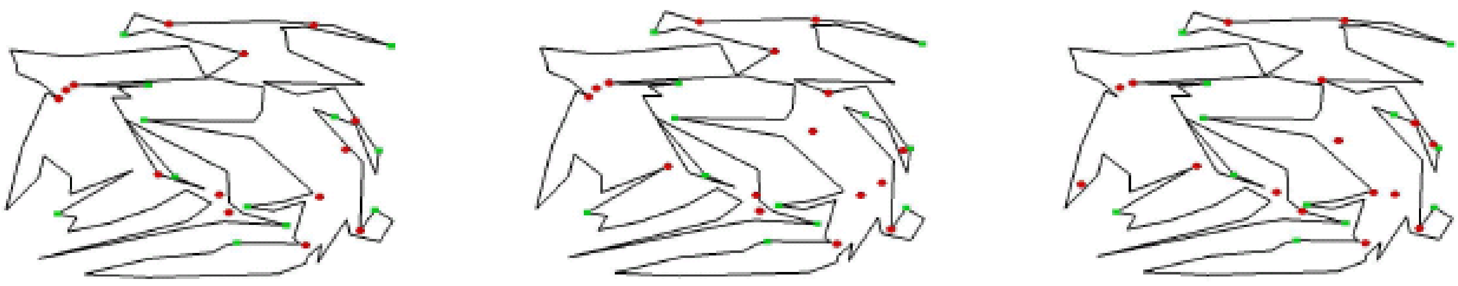
(d) 14 guards (e) 14 guards (f) 13 guards

A_1 A_2 A_{11}

Examples: n=100



(g) 16 guards (h) 16 guards (i) 15 guards

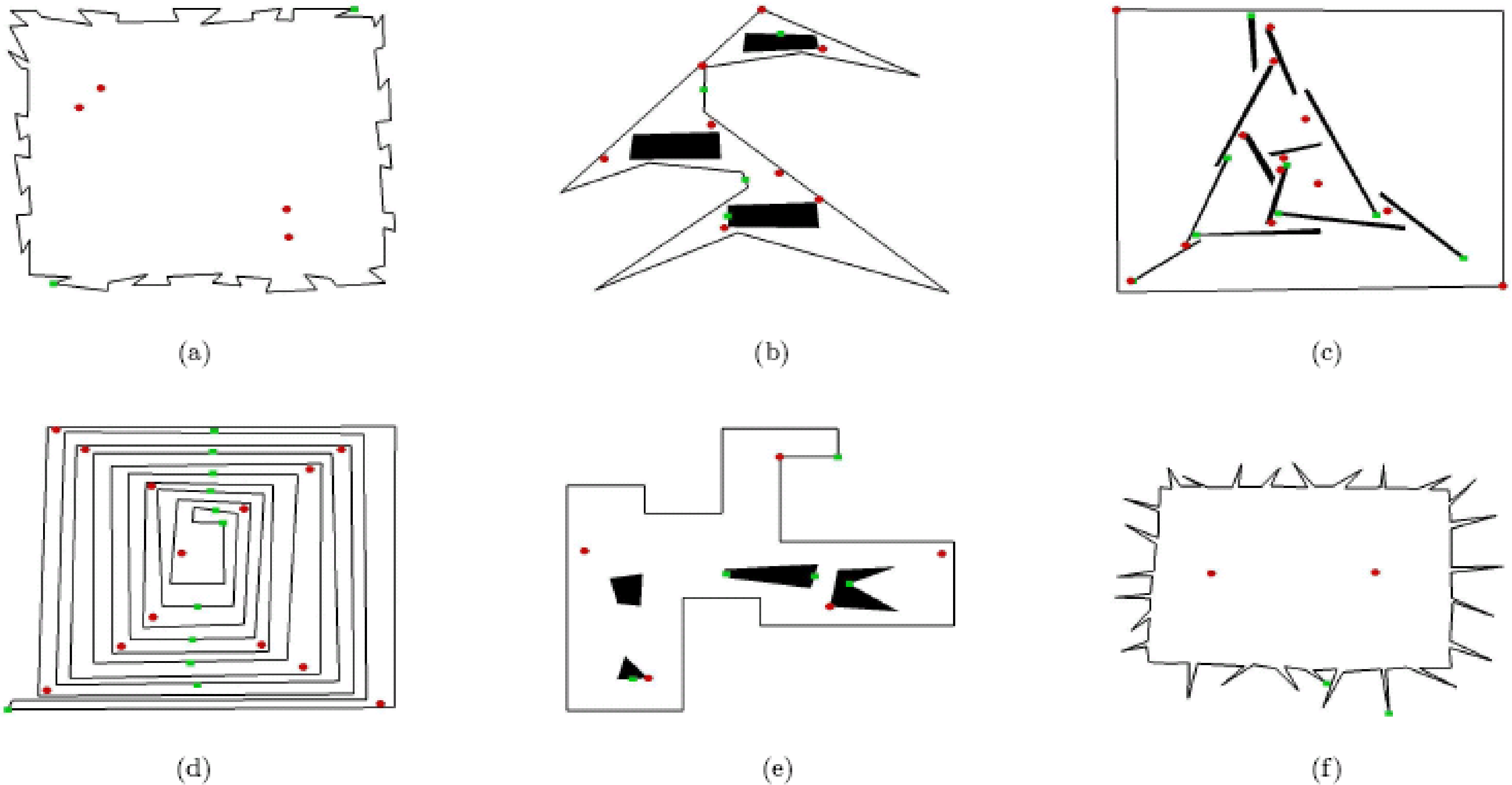


(j) 14 guards (k) 16 guards (l) 16 guards

A_1 A_2 A_{11}

More Examples

A_1



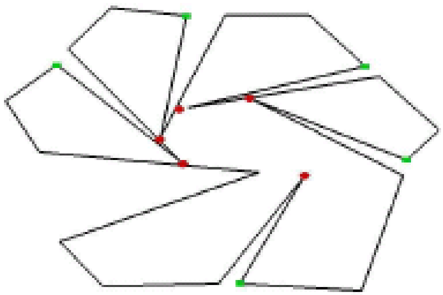
(a) (b) (c)

(d) (e) (f)

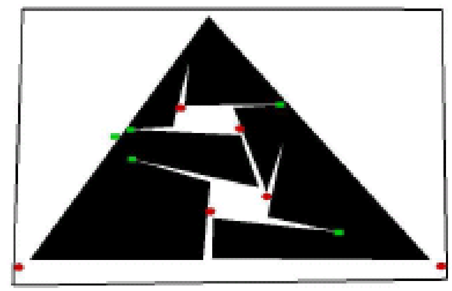
Spike box

More Examples

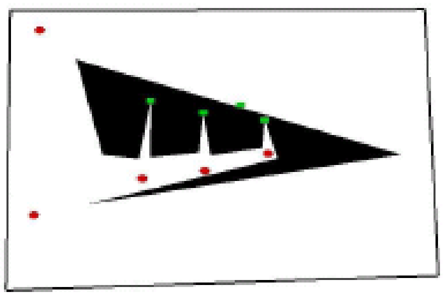
A_1



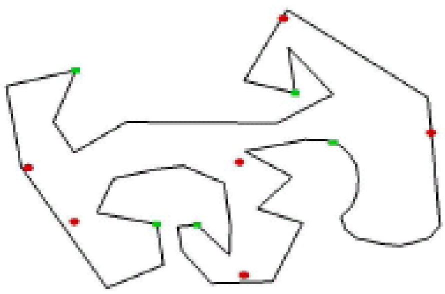
(g)



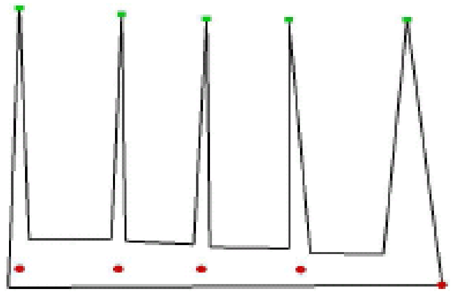
(h)



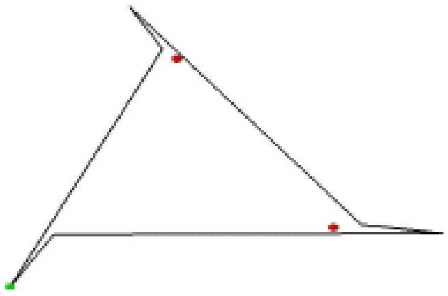
(i)



(j)



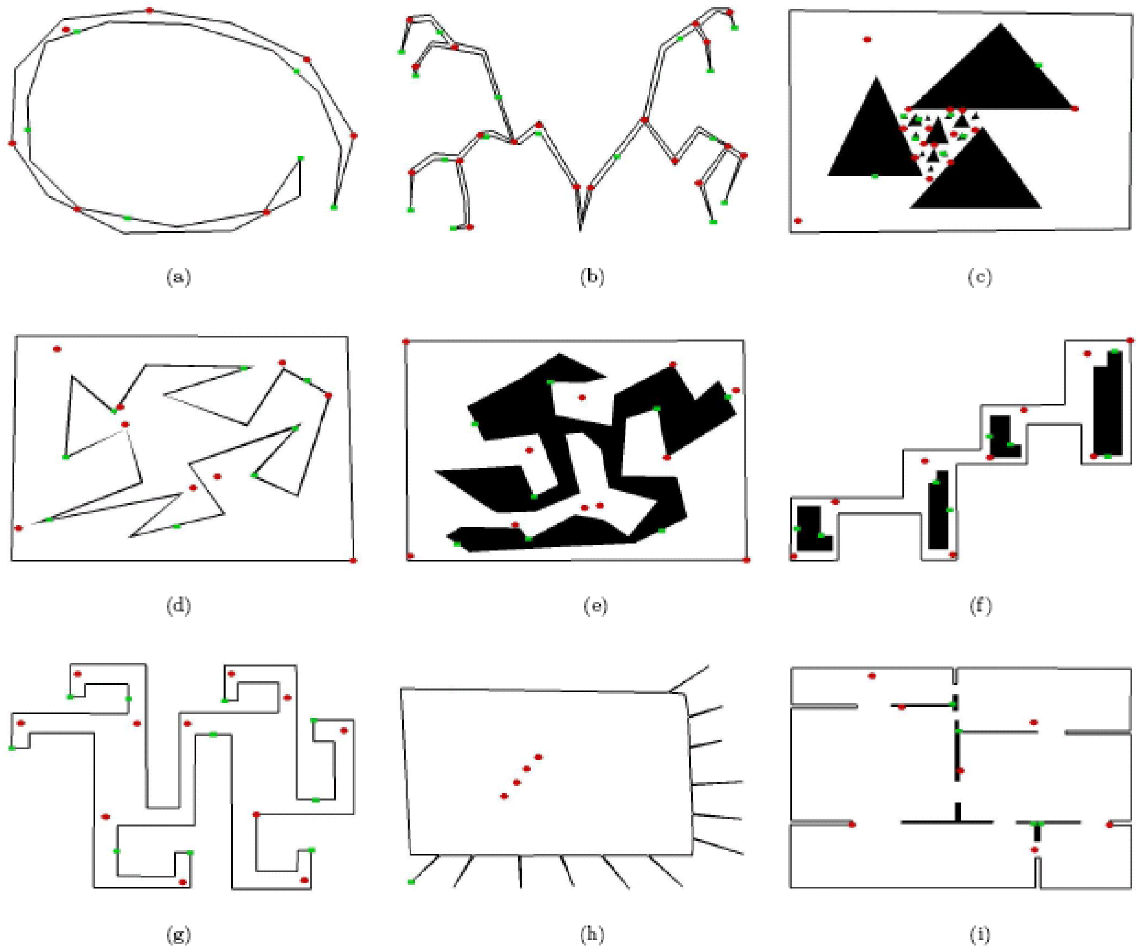
(k)



(l)

More Examples

A_1



Comparison of Heuristics

Results on 40 polygons:

	A_1	A_2	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
K	0.7	0.47	1.47	1.6	1.3	1.22	0.9	0.83	1.75	0.48	0.5	1.64	3.33
M	0.10	0.06	0.22	0.22	0.16	0.29	0.13	0.14	0.41	0.08	0.09	0.27	0.69
Q	16	17	11	11	10	10	11	12	8	15	15	9	8
B	40	40	40	40	40	40	40	30	29	39	38	39	30

Table 1: Results obtained with our heuristics on 40 input sets.

K - average **excess** = *number* of guards *more* than the *min* guard number over all heuristics

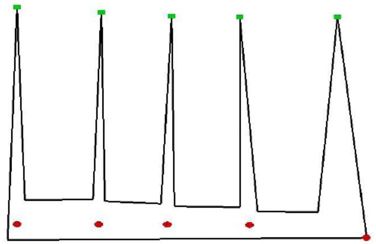
M – average **relative excess** (relative to min)

Q - number of times (out of 40) the guarding obtained with the heuristic was the **best** among all heuristics

B - number of completed tests

Comparison of Heuristics

PN	V	RV	H	VH	RVH	A_1	A_2	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	LB
1	24	10	0	0	0	5	4	6	5	4	5	5	4	5	4	4	3	5	2
2	9	3	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	12	7	3	12	12	8	7	8	8	7	8	8	7	6	7	7	8	X	4
4	44	20	0	0	0	13	13	12	12	13	14	13	13	11	12	12	11	20	11
5	83	52	0	0	0	2	2	2	2	2	10	2	2	X	X	X	3	22	2
6	33	19	0	0	0	2	2	3	2	3	4	2	2	4	2	2	4	4	2
7	24	6	0	0	0	5	5	5	5	5	5	5	5	6	5	5	5	6	5
8	4	0	12	60	48	13	13	14	14	16	12	16	18	15	9	9	14	X	8
9	17	7	0	0	0	3	3	5	4	4	3	4	3	4	3	3	3	4	3
10	5	2	1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	X	2
11	6	2	1	5	5	4	4	4	4	4	4	3	3	4	4	4	5	X	2
12	4	0	13	48	44	15	13	17	17	16	15	18	16	14	13	X	15	X	7
13	16	6	4	16	5	5	5	7	9	8	6	6	7	6	6	6	7	X	5
14	6	3	0	0	0	2	2	2	2	2	2	2	2	2	2	2	3	X	1
15	4	0	1	20	14	6	6	6	6	6	6	6	6	6	6	6	7	X	5
16	4	0	1	13	9	5	5	5	5	5	5	5	5	5	5	5	5	X	4
17	15	8	0	0	0	5	5	5	5	5	5	5	5	5	5	5	5	5	5
18	49	24	0	0	0	6	7	6	7	7	7	6	6	28	5	5	X	12	4
19	100	46	0	0	0	13	12	16	16	15	14	13	X	X	13	13	15	X	11
20	100	47	0	0	0	18	15	19	19	18	15	15	X	X	16	16	18	20	12
21	100	44	0	0	0	16	15	18	17	16	16	16	X	X	16	16	19	19	13
22	100	50	0	0	0	14	14	15	15	15	15	14	X	X	14	14	16	16	9
23	100	45	0	0	0	18	18	18	19	18	17	16	X	X	17	17	17	19	13
24	100	55	0	0	0	16	16	18	18	18	16	16	X	X	15	15	16	21	14
25	100	49	0	0	0	18	17	18	18	18	18	18	X	X	17	17	22	22	14
26	100	46	0	0	0	17	16	19	18	18	17	18	X	X	16	16	17	19	13
27	100	49	0	0	0	12	12	15	16	13	15	12	X	X	14	14	18	16	11
28	100	52	0	0	0	14	16	15	15	17	17	15	X	X	16	16	16	19	12
29	10	3	0	0	0	2	1	2	2	1	2	2	2	2	2	2	2	2	1
30	10	3	0	0	0	1	1	1	1	1	2	1	1	2	1	1	2	1	1
31	10	4	0	0	0	2	2	2	2	2	2	2	2	3	2	2	3	2	2



Comparison of Heuristics

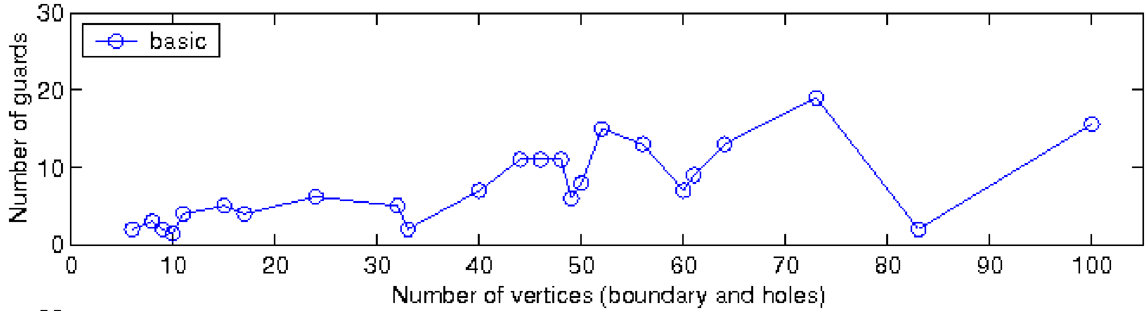
PN	A_1	A_2	A_{11}	I_1	I_2	I_3	min ratio
1	5	4	4	2	1	1	2
2	2	2	2	2	2	X	1
3	8	7	7	4	2	4	1.75
4	13	13	12	11	8	10	1.09
5	2	2	X	2	2	1	1
6	2	2	2	2	2	1	1
7	5	5	5	5	5	X	1
8	13	13	9	8	1	8	1.125
9	3	3	3	3	3	2	1
10	3	3	3	2	1	1	1.5
11	4	4	4	2	2	2	2
12	15	14	13	7	2	6	1.85
13	5	5	6	5	3	3	1
14	2	2	2	1	1	X	2
15	6	6	6	5	5	3	1.2
16	5	5	5	4	4	2	1.25
17	5	5	5	5	5	1	1
18	6	6	5	4	4	X	1.25
19	13	12	13	11	11	6	1.09
20	18	15	16	11	12	8	1.25
21	16	15	16	13	12	7	1.15
22	14	15	14	9	9	5	1.55
23	18	18	17	13	13	5	1.3
24	16	16	15	14	14	7	1.07
25	18	17	17	14	14	8	1.21
26	17	16	16	13	11	7	1.23
27	12	12	14	11	11	5	1.09
28	14	16	16	12	11	5	1.16
29	1	1	1	1	1	X	1
30	1	1	1	1	1	X	1
31	2	2	2	2	2	1	1
32	2	2	2	2	2	1	1
33	1	1	1	1	1	1	1
34	2	2	2	2	2	1	1
35	1	1	1	1	1	X	1
36	1	1	1	0	1	1	1
37	1	1	1	1	1	1	1
38	2	2	2	2	2	X	1
39	10	9	9	8	8	3	1.125
40	8	8	8	7	7	3	1.14
41	9	9	9	8	8	3	1.125
42	6	6	8	6	6	5	1
43	6	6	7	6	6	1	1
44	10	10	10	8	8	3	1.25
45	8	8	8	8	7	4	1
46	8	9	8	8	8	3	1
47	6	6	6	6	6	3	1

Comparison of Heuristics

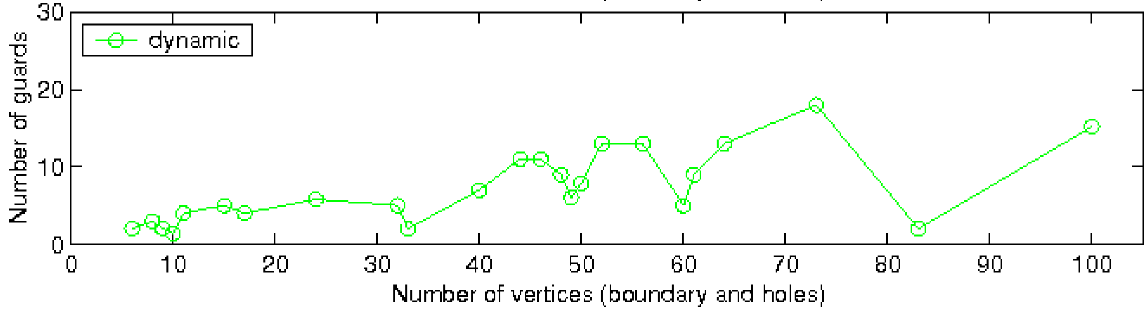
PN	V	RV	H	VH	RVH	A_1	A_2	A_{11}
41	24	11	0	0	0	7	7	6
42	73	40	0	0	0	19	18	18
43	4	0	17	48	48	15	12	14
44	4	0	1	57	26	9	9	9
45	4	0	1	44	26	11	9	8
46	20	8	4	24	20	9	9	10
47	24	10	8	32	32	13	13	13
48	46	21	0	0	0	11	11	11
49	40	19	0	0	0	10	10	10
50	28	12	4	32	24	7	5	6
51	40	120	0	0	0	4	4	4
52	200	101	0	0	0	35	X	33
53	200	95	0	0	0	31	X	31
54	200	100	0	0	0	31	X	27
55	200	100	0	0	0	28	X	31
56	200	104	0	0	0	31	X	31
57	200	97	0	0	0	26	X	26
58	200	98	0	0	0	26	X	27
59	200	98	0	0	0	30	X	31
60	200	98	0	0	0	26	X	26
61	200	99	0	0	0	31	X	30

Number of Guards vs. Number of Vertices

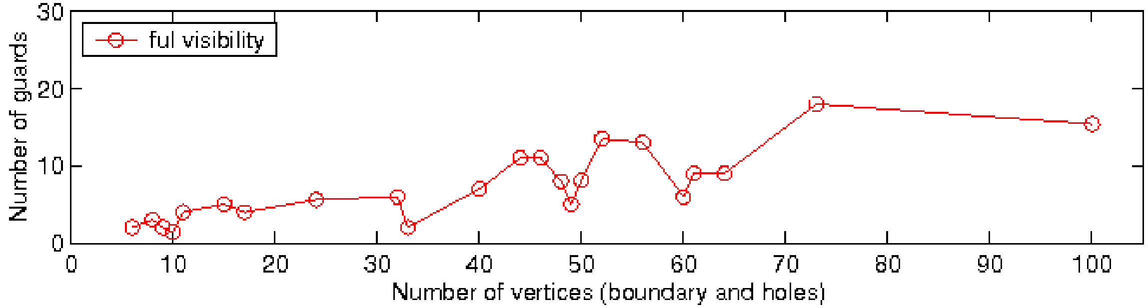
A_1



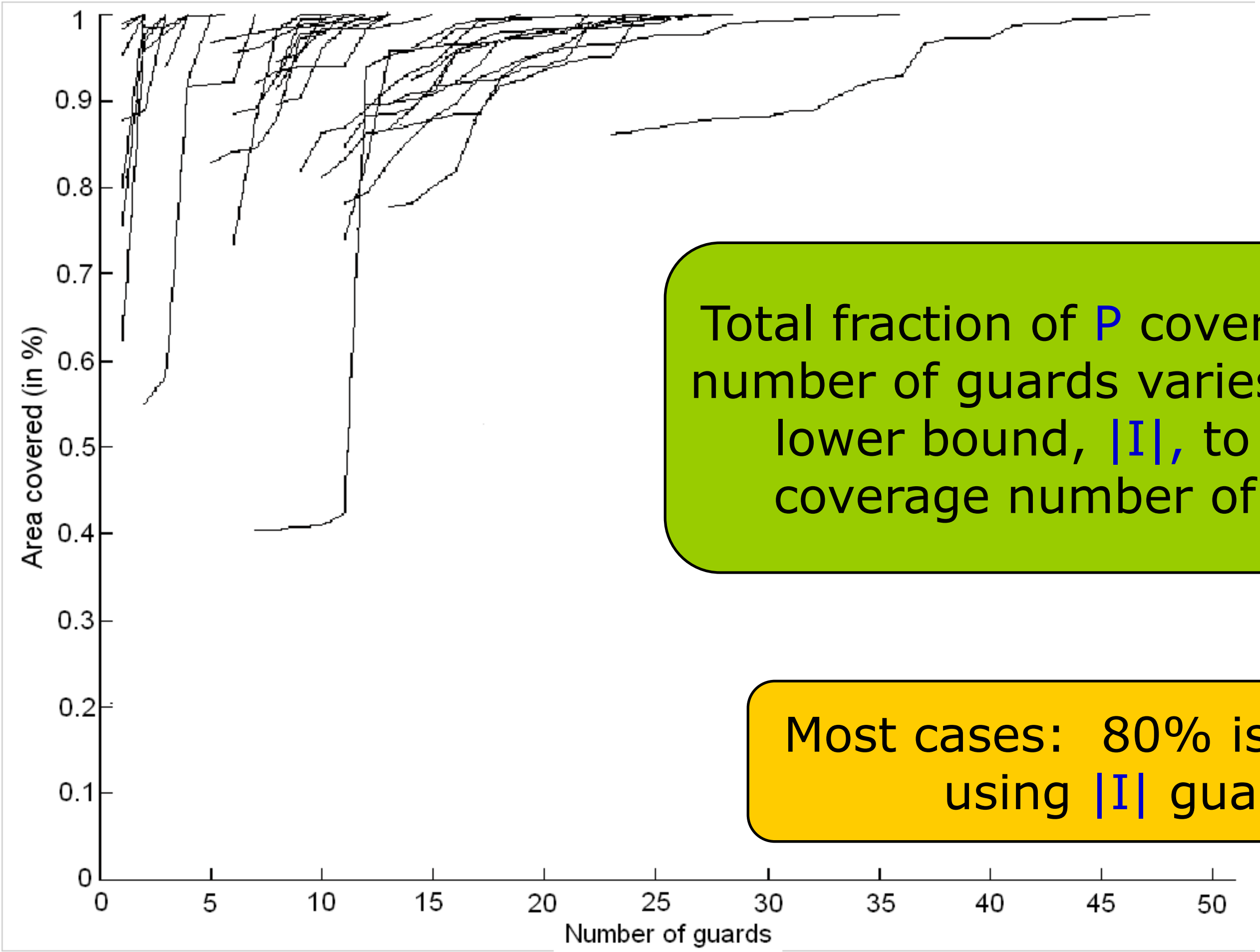
A_2



A_{11}



Early Termination: Partial Covering



Total fraction of P covered as the number of guards varies from the lower bound, |I|, to the full coverage number of guards

Most cases: 80% is covered using |I| guards

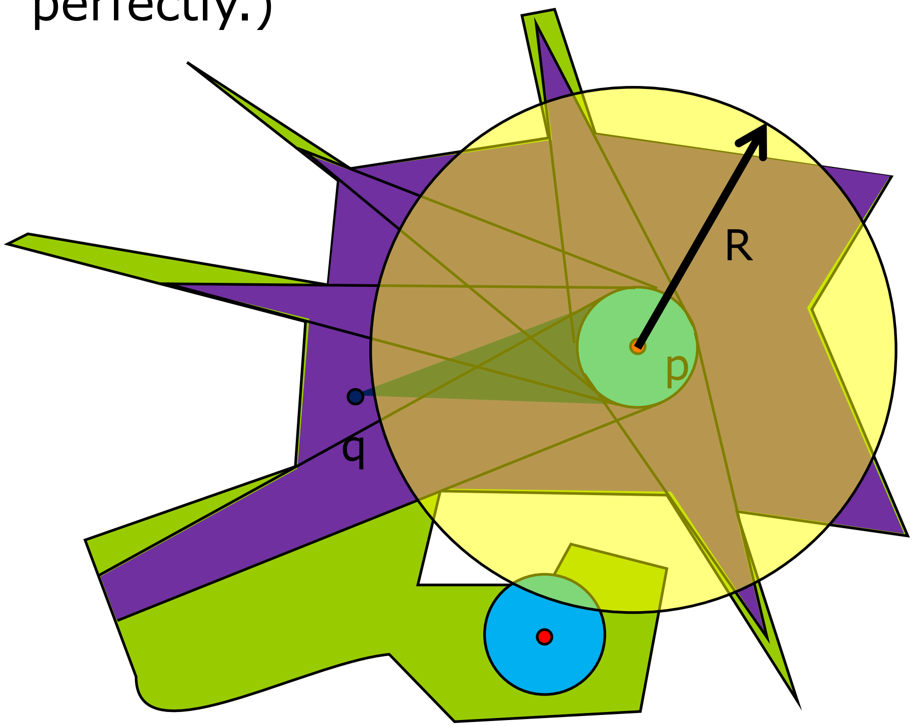
Conclusion

- Extensions:
  - Visibility constraints (view distance, good view angles, robust coverage)
  - Terrain coverage (2.5D)
  - 3D
- Open:
  - Any approx algorithm (better than n/3-approx) for unrestricted guards
  - O(1)-approx for vertex/grid guarding simple polygons
  - Characterization of polygons for which our heuristics perform well (provably well)?

Robust Guards

Issue: Even if we computed exactly a minimum cardinality set of guards, could we know with confidence the domain is really guarded?

Guards may not be placed exactly. (Human guards don't usually stand exactly still, and cameras/sensors cannot be placed perfectly.)



Model: When a guard is placed at p, it will actually reside at some point within a disk, $B_\varepsilon(p)$, of radius ε

In order for q to be "seen" by guard p, it must be able to see the guard no matter where it is within the disk $B_\varepsilon(p)$

Bounded radius, R, of vision

Robust Guards: New Approx Bound

Theorem: There is a PTAS for computing a min # of robust, radius-bounded guards in a polygonal domain (with holes), assuming R/ε is bounded, and a poly-size set G of candidate guard locations is given.

One option for G: use a set L of $O(\lambda \log^2 \lambda)$ *landmarks*, as in [AEG08], and then guarantee at least $(1-\varepsilon_1)$-fraction of the area is seen.

$\lambda = (g_{opt} / \varepsilon_1) \log h$ (h = # holes)

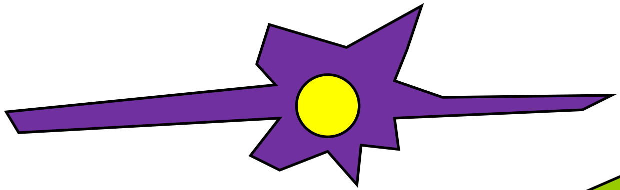
[AEG08] also give randomized greedy algorithm that, whp, computes $O(g_L \log \lambda)$ guards to cover L, where $g_L \leq g_{opt}$ is opt # of guards to cover L

Method: m-guillotine optimization: Convert any OPT to an m-guillotine version; apply DP to optimize

What is Needed for PTAS to Apply

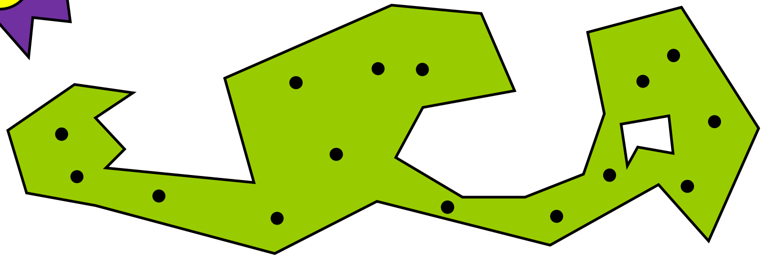
Suffices: Visible regions, VP(g), from candidate guard locations $g \in G$ have area(VP(g)) ≥ c diam^2(VP(g)), for some c. (e.g., each VP(g) contains a disk of radius Ω(diam(VP(g)))

Special Case: Bounded radius visibility in polyominoes



Another Sufficient Model:

Sample points S in P.
Guards placed at subset of S.
Guards must see all of S: Problem is **Dominating Set** in VG(S)



If samples S are δ-*well dispersed* (e.g., no disk of radius δ has more than O(1) samples of S), and guards have visibility radius R, with R/δ bounded, then PTAS also applies

Minimum Dominating Set:
best approx in general is log-approx
PTAS for planar graphs, UDG
APX-complete for degree-B, B≥3

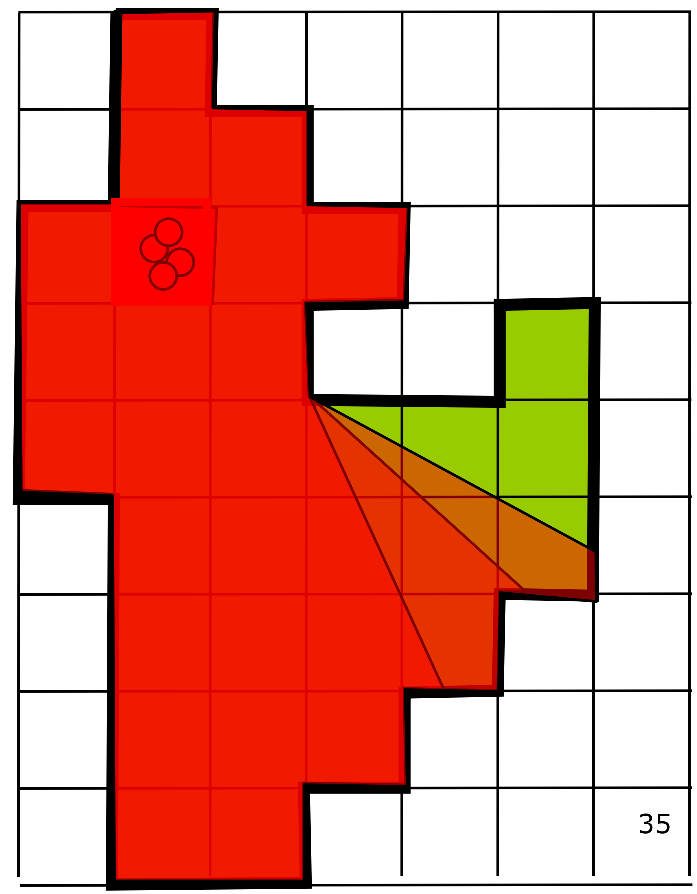
Here, the graph VG(S) is not planar, not UDG, but has bounded degree, depending on R/δ

Guarding Polyominoes

[Irfan, Iwerks, Kim, M]

- Polyomino: simply connected union of m integral unit squares (pixels) – “pixel polygon”

- Models of pixel guards:
  (1) Point guards
  (2) Pixel guards
  (3) Robust (pixel) guards:
  Strong visibility: only those points that are seen from *any* point within the pixel are seen

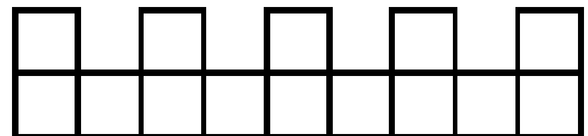


Guarding Polyominoes

Art Gallery Thm:

(1) ceil((m-1)/3) point guards suffice and are sometimes necessary

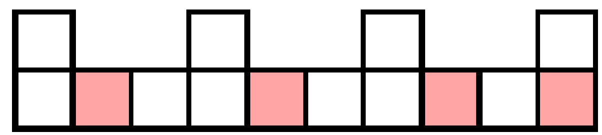
Point Guards



(2) ceil((m-1)/3) pixel guards suffice and ceil((m-1)/4) are sometimes necessary

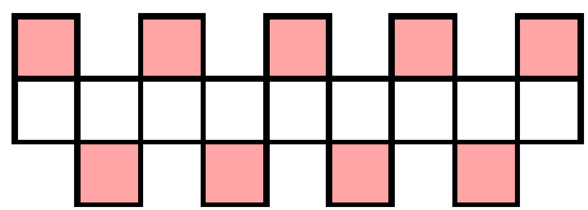
OPEN: Close the gap!

Pixel Guards



(3) floor(m/2) robust guards suffice and are sometimes necessary: Simple coloring argument: 2-color the grid of pixels.

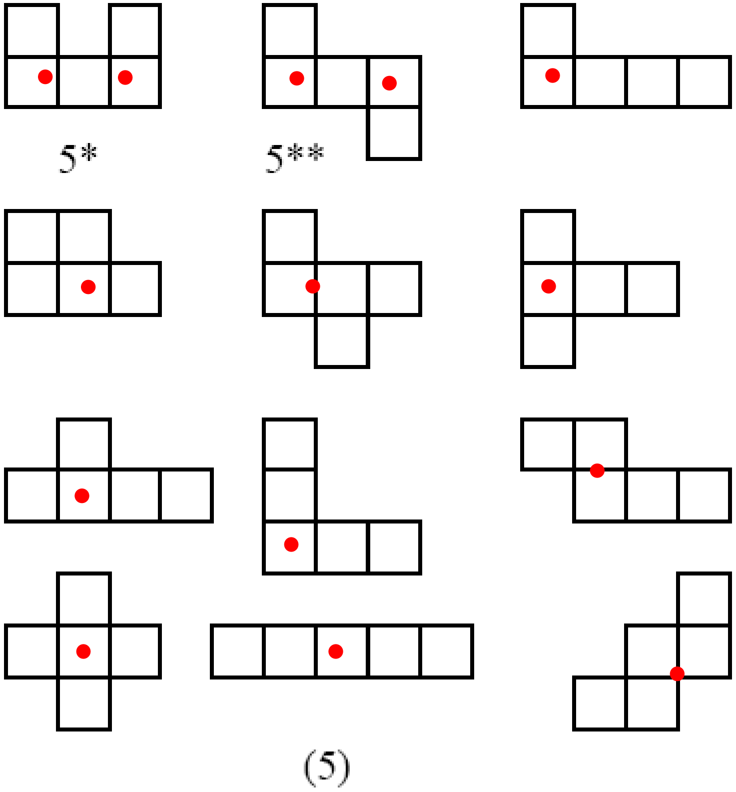
Robust Pixel Guards

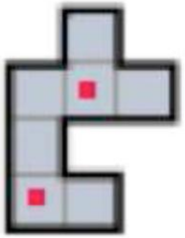


NP-hardness: Computing the guard number in polyominoes is NP-hard

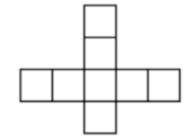
Examples of pentominoes

Each requires just one point guard, *except* 5* and 5**

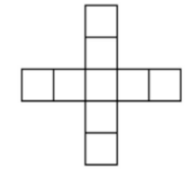




Point Guards in Polyominoes



8*



9*

Claim 1 *Let P be an m-pixel polygon where $m \geq 2$. Then there exists a pixel p that can be removed from P yielding P' such that P' is simply connected.*

Claim 2 *Let P be any (8) besides 8^*. Then we can decompose P into two connected pixel subpolygons P_1 and P_2 such that either $|P_1| = |P_2| = 4$ or $|P_1| = 3$ and $|P_2| = 5$.*

Corollary 1 *If P is any (9) besides 9^*, then we may decompose P into two subpolygons P_1 and P_2 such that either $|P_1| = 3$ and $|P_2| = 6$ or $|P_1| = 4$ and $|P_2| = 5$. Also, any (10), P, is decomposable into two pixel subpolygons P_1 and P_2 such that either $|P_1| = 3$ and $|P_2| = 7$, $|P_1| = 4$ and $|P_2| = 6$, or $|P_1| = |P_2| = 5$.*

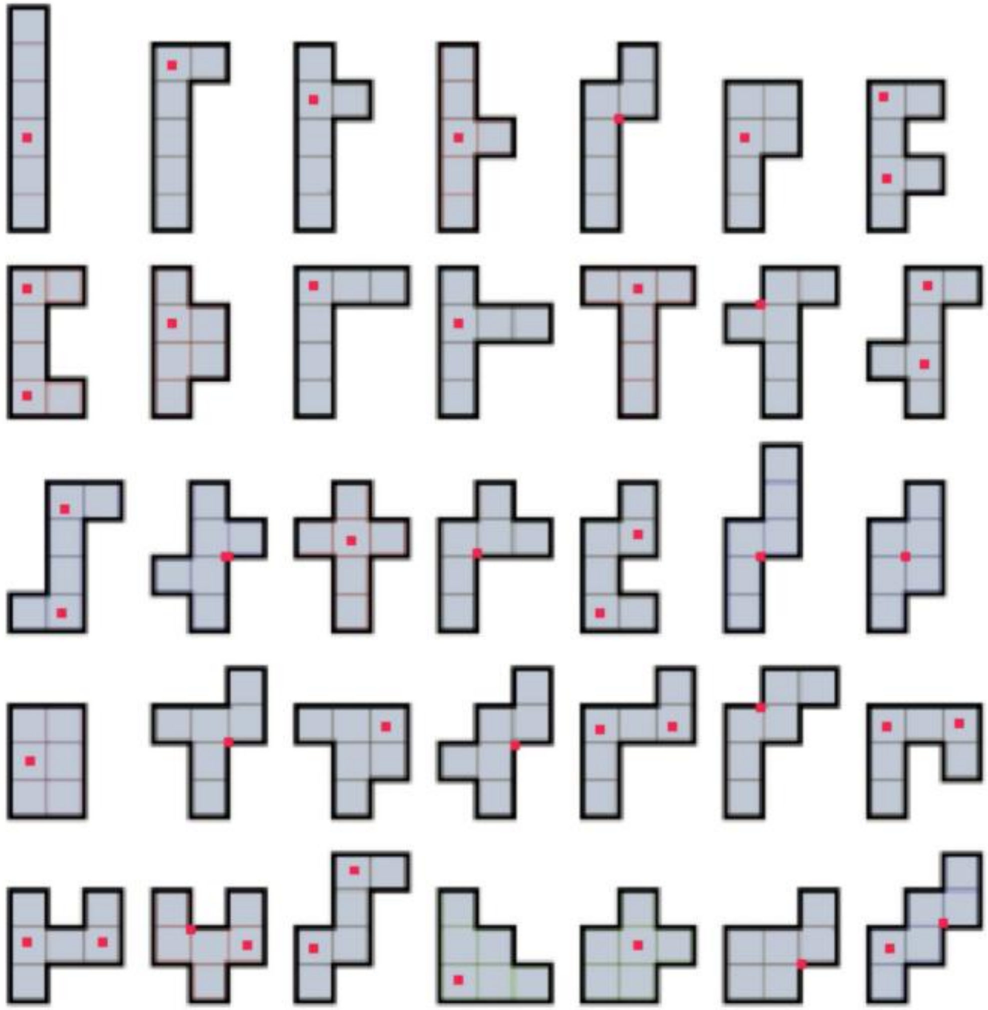
Claim 3 *For any m-pixel polygon P where $m = 1, 2, 3$, or 4, one point guard is sufficient to guard P. For any m-pixel polygon P where $m = 5, 6, 7$, two point guards are sufficient to guard P.*

Claim 4 *For any m-pixel polygon P with $m \geq 3$, pixel subpolygons $S_i \in \{(3), (4), ((5)/5^*, 5^{**}), (6), (7), 8^*, 9^*\}$ $(i = 1, 2, 3, \ldots, f)$ can be removed from P yielding connected pixel polygons $P_1, P_2, \ldots, P_f$ where P_i is the connected pixel polygon remaining after removing the i^{th} pixel subpolygon from P. Also, P_f may contain 0, 1, or 2 pixels.*

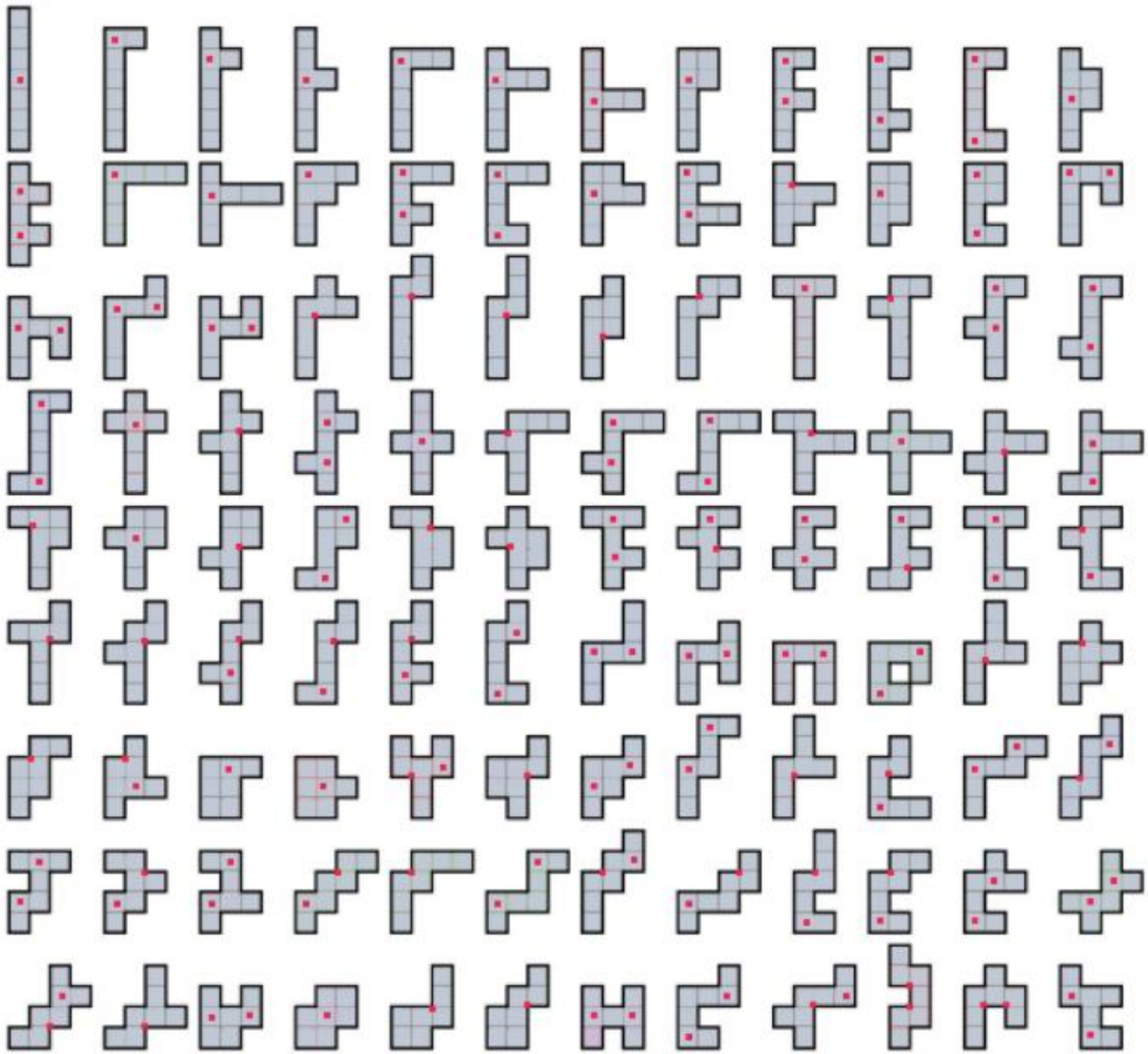
Corollary 2 $\lceil \frac{m}{3} \rceil$ *point guards is sufficient to guard an m-pixel connected polygon P.*

Actually, ceil((m-1)/3)

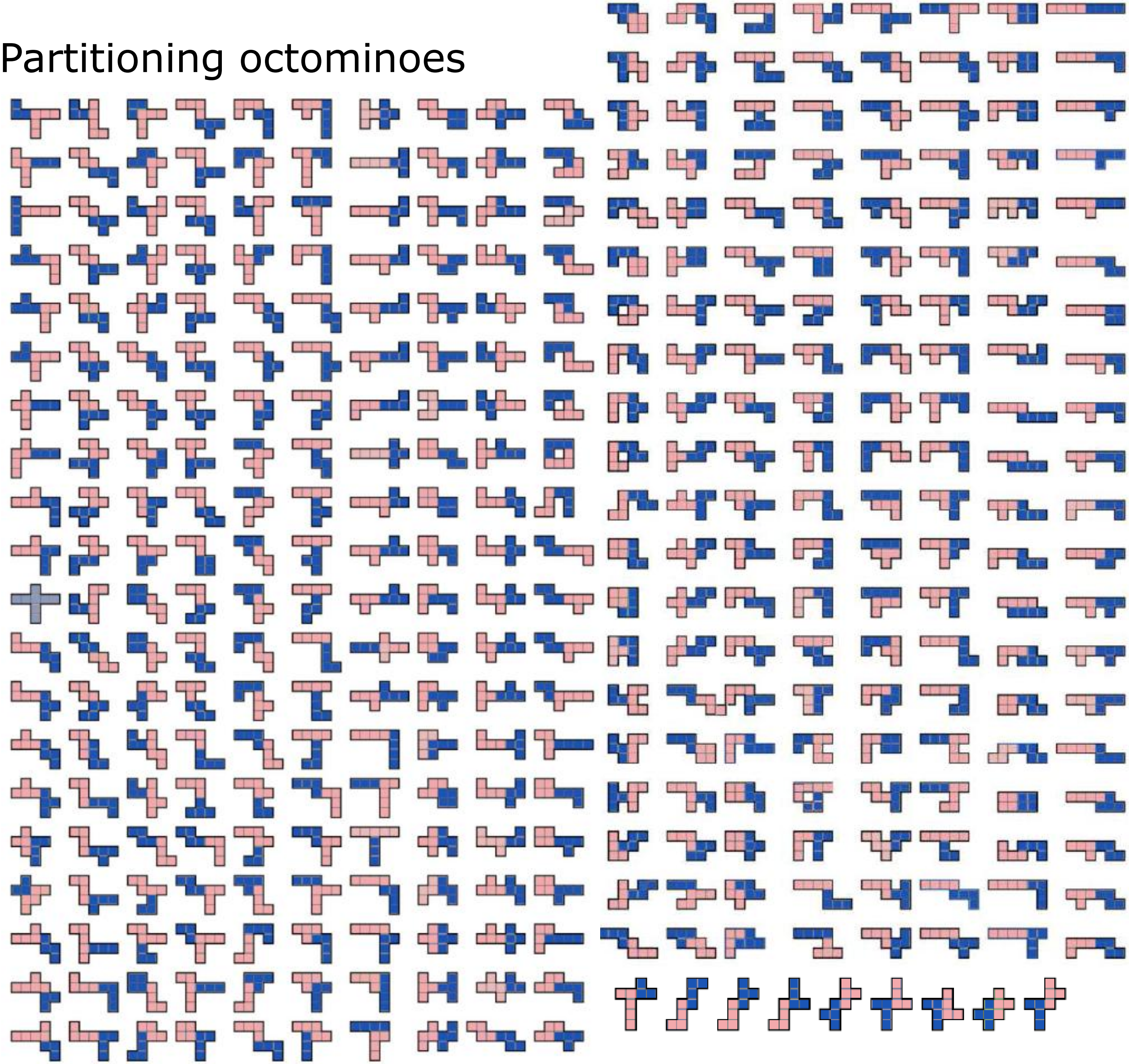
Claim: Any hexomino (m=6) can be guarded with 1 or 2 points.



Claim: Any heptomino (m=7) can be guarded with 1 or 2 points.

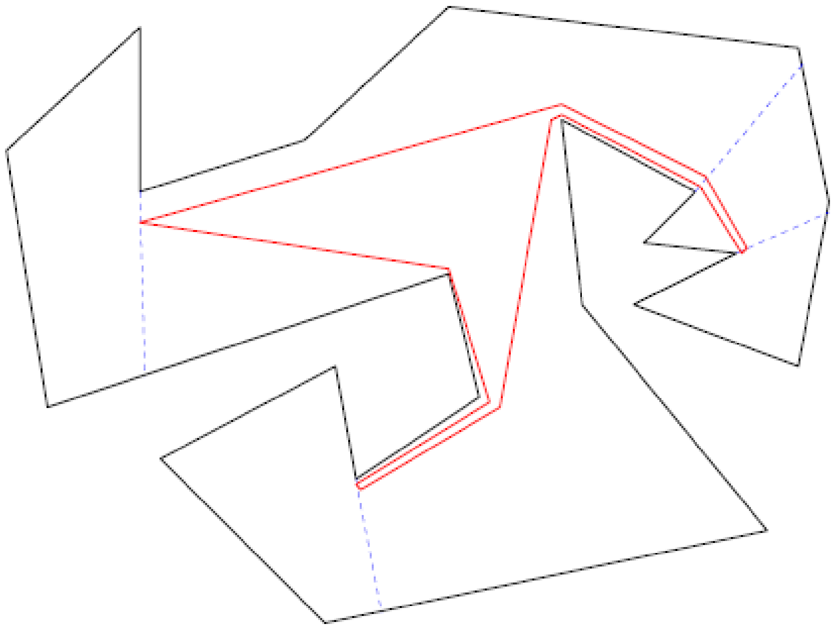


Partitioning octominoes



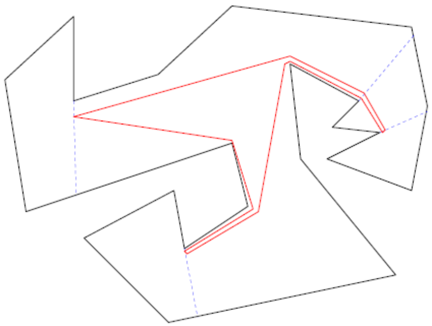
Mobile Guards

Watchman Route Problem



Find a shortest tour for a guard to be able to see all of the domain

Watchman Route Problems



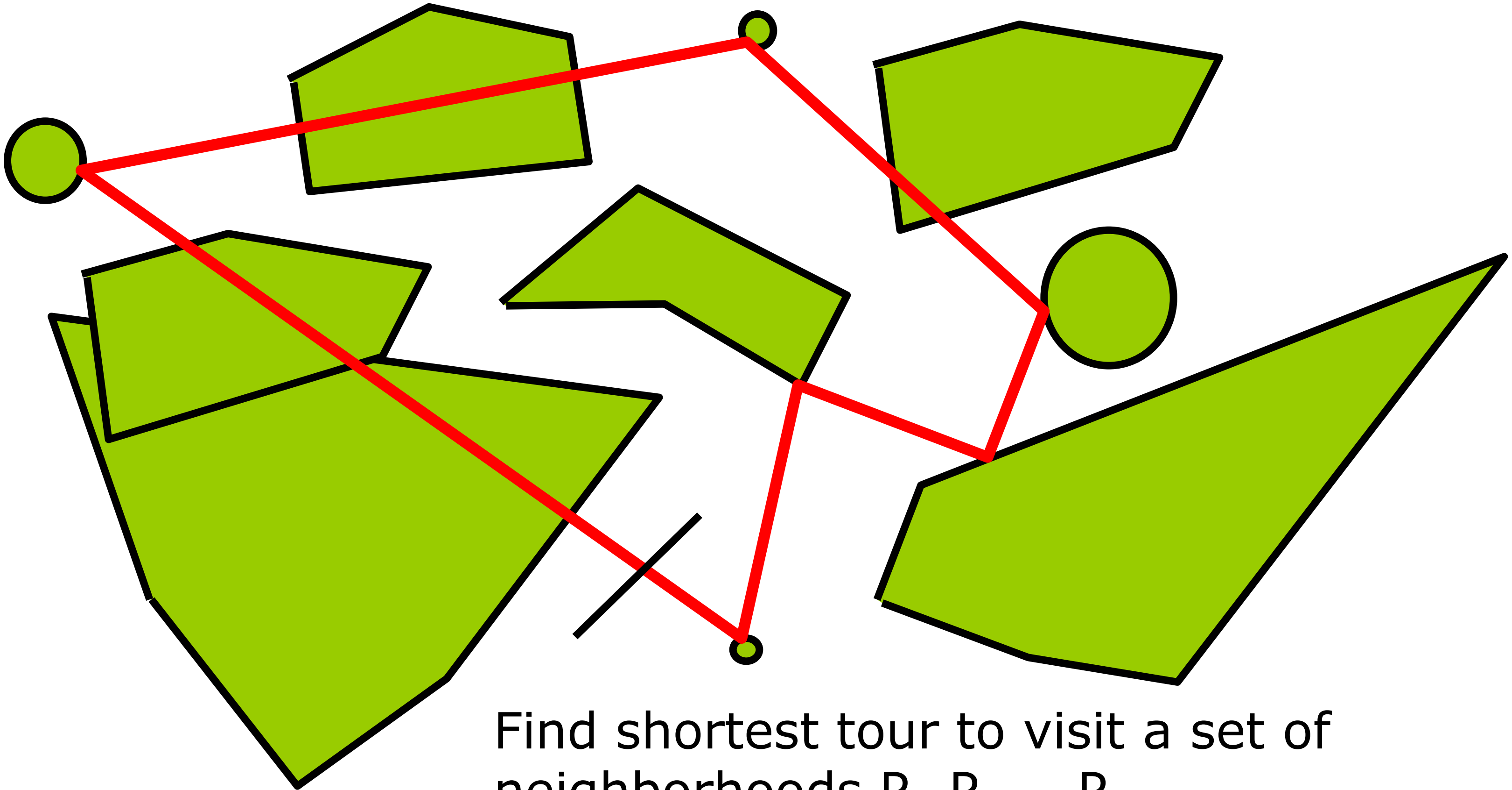
- Closely related to TSPN: visit VP(p), for all p in P
- Poly-time in simple polygons [CN,DELM]
  - Best time bound: $O(n^3 \log n)$ [DELM]

- NP-hard in polygons with holes
  - No approx algorithm known in general!
  - Rectilinear visibility: O(log n)-approx [MM'95]
  - NEW: For fat obstacles, PTAS to see at least one point on the boundary of each obstacle
- 3D: Depends on 3D TSPN [ADDFM]

Q: Approx for planar domain, standard visibility?

Q: Approx for guard on a terrain surface?

TSPN: TSP with Neighborhoods

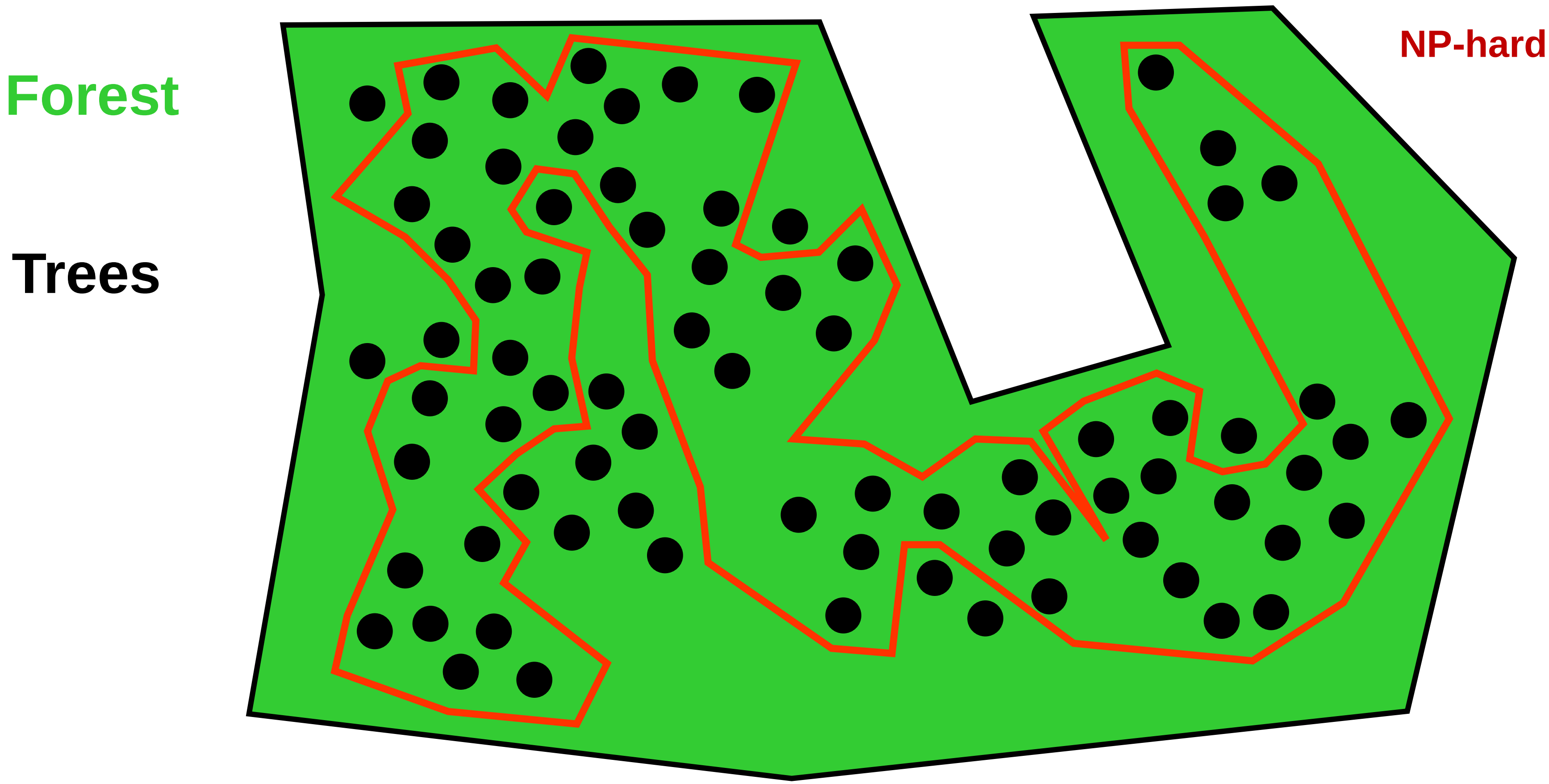


Find shortest tour to visit a set of neighborhoods $P_1, P_2, \ldots, P_n$

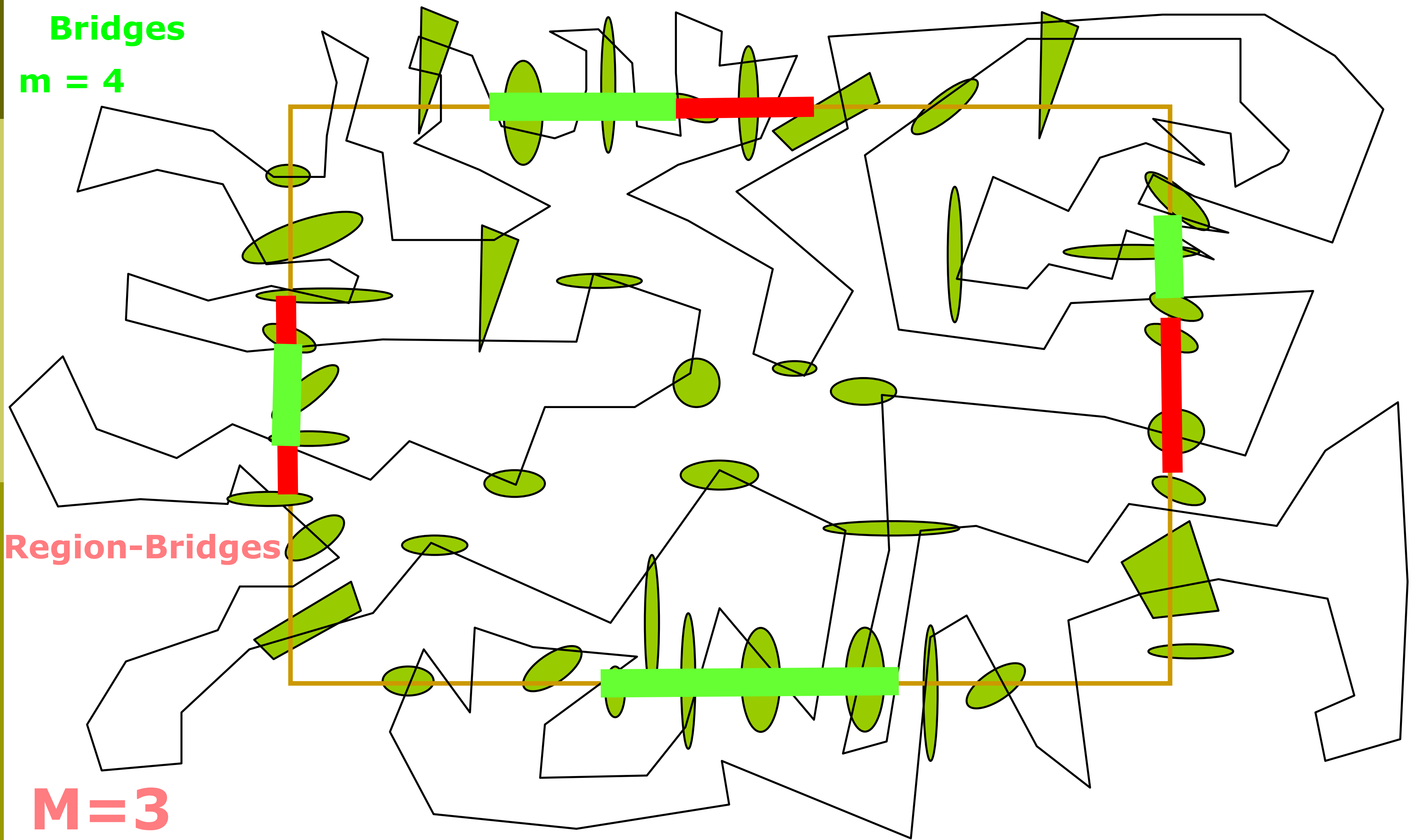
Watchman: How to
"See the Forest for the Trees"

Recent result: Can apply also to yield PTAS for watchman route among fat obstacles

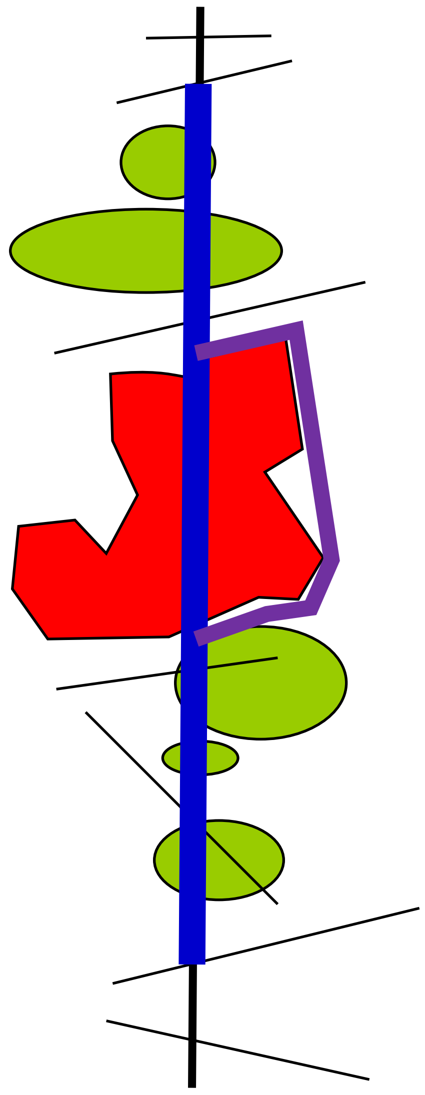
Fat obstacles: Prove m-guillotine PTAS applies to geodesic metric



TSPN Subproblem: A Window into OPT



TSPN with Obstacles: Key Issue



Bridge (as in m-guillotine method)

Obstacle

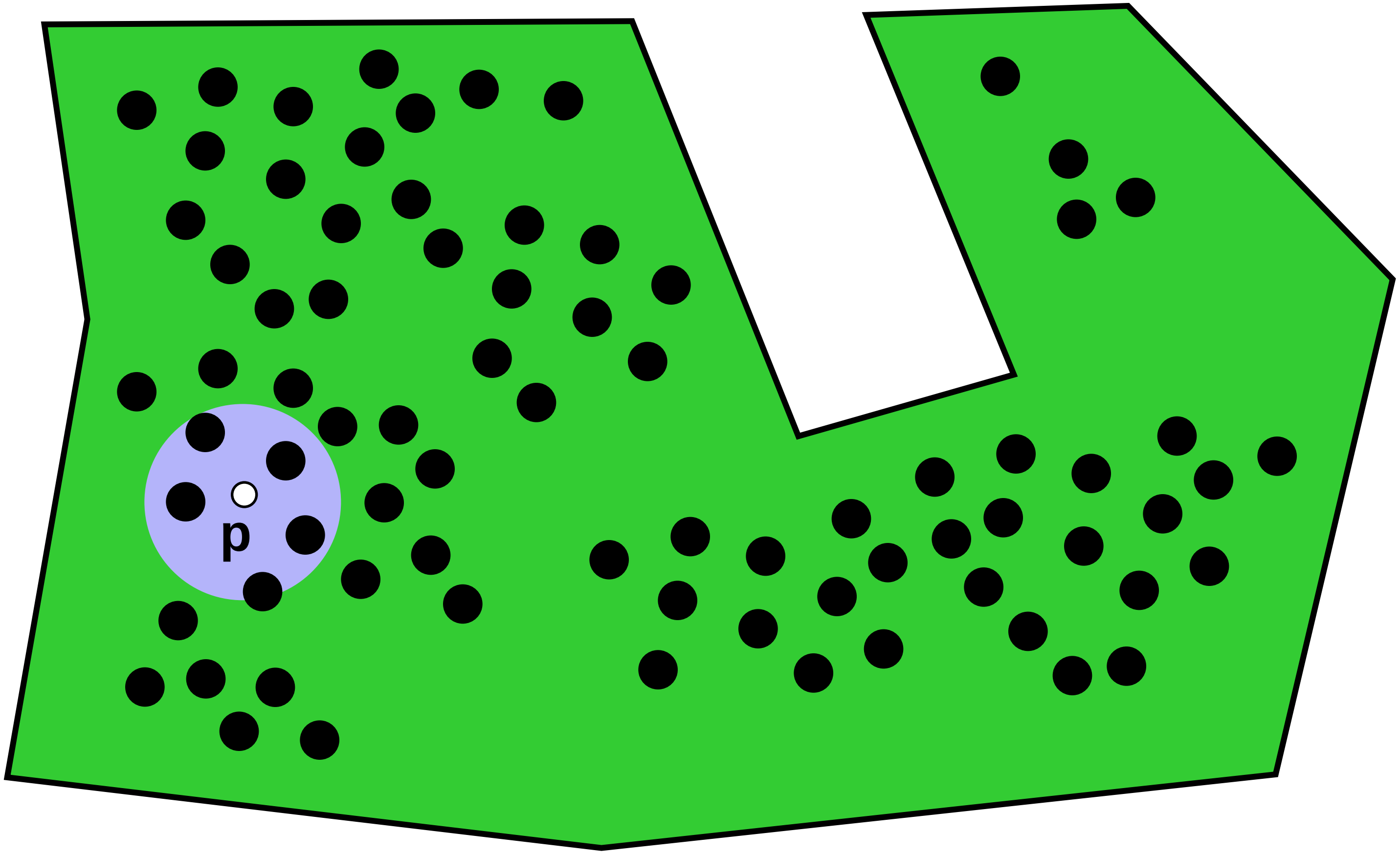
Detour (needed to keep the Bridge connected)

Sufficient: Obstacles are ***fat*** : then the detours to keep bridge connected cause only a constant-factor dilation to bridge length, which is charged off

Forest Assumptions

Either: (1) limited view distance

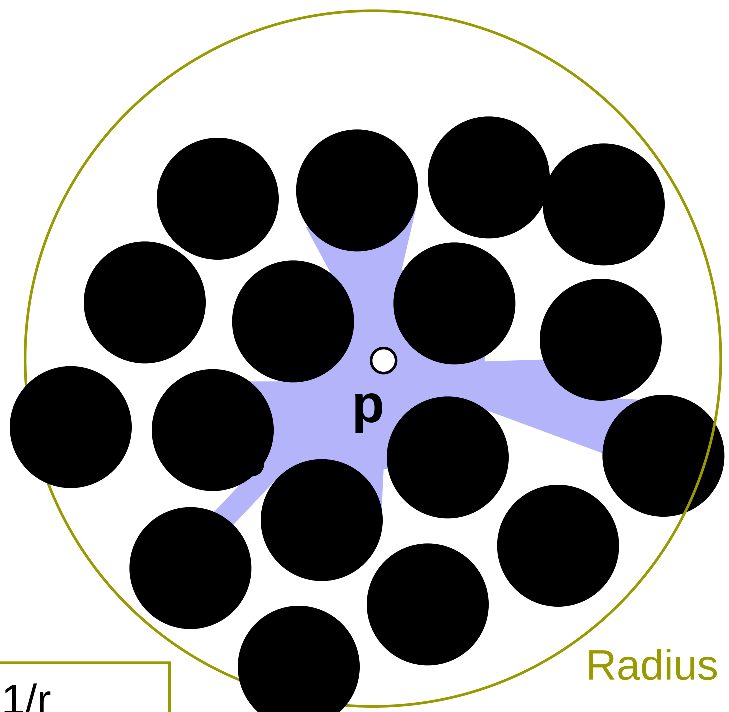
Require robot to get within distance ***R*** of a point ***p*** in order to see it



Forest Assumptions

Or: (2) forest is dense enough (e.g., maximal packing) so that the visibility region from a point deep inside the forest is a **fat** (star-shaped) region.

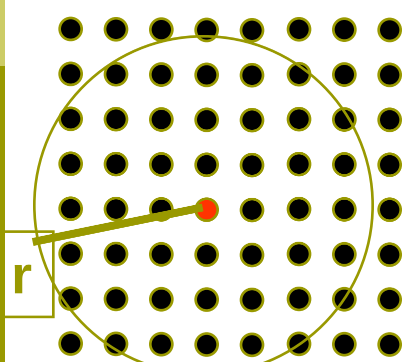
Time: $O(n^{O(R)})$



Dark Forest Conjecture: For R < const, there exists a dark point p

Recently shown!: R < const [Dumitrescu and Jiang, 2009]

$R < 2*10^{108}$



Dark if tree radius > 1/r

Related to Polya's Orchard Problem

Olber's paradox [1826]